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LETTER TO THE EDITOR

Quantum magnetic confinement in a curved two-dimensional electron gas

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Abstract. The ability to produce deliberately shaped or curved two-dimensional electron gases in semiconductors using recent developments in technology, for example regrowth of III–V semiconductors on patterned or etched substrates, opens the possibility of investigating not only the behaviour of electrons in a curved quasi-two-dimensional space, and the effects of varying that curvature, but also presents a novel way of investigating electron transport properties in a non-uniform transverse high magnetic field. It is shown that a semi-infinite two-dimensional electron gas subjected to a non-uniform magnetic field has, in addition to current-carrying edge states, one-dimensional states which lie within the interior of the gas, which also have a finite dispersion, an effect which may be used to create quantum wires or other structures. It is also shown that, in the absence of a magnetic field, curvature of the two-dimensional electron gas gives rise to a potential variation which is inversely proportional to the square of the radius of curvature, an effect which may also be used to confine the electronic motion to one dimension.

The transport properties of two-dimensional electron gases (2DEG) subjected to a perpendicular magnetic field have been the focus of a great deal of research in recent years [1–4]. One particular property of interest is the conductance quantization, which occurs as a result of the one-dimensional nature of the conduction channels in such systems, and the associated quantum hall effects (QHE), which are thought to arise due to the suppression of backscattering [2, 3]. Alongside this has been the move towards attainment of quantum systems of ever fewer dimensions, namely quantum wires [5, 6] and quantum dots [7], produced by electrostatic confinement [8]. In addition, there has been a growth in interest, both theoretical and experimental, in two-dimensional electron transport in non-uniform magnetic fields [9–11], produced via the use of e.g. superconductor–semiconductor interfaces [12].

In this letter a method is introduced by which it is possible to investigate both electron transport in a non-uniform high magnetic field, and the effects of varying, in addition to the number of dimensions they have available for motion, the topology of the space in which the electrons are confined. This method is illustrated through the use of a simple example structure, exhibiting most of the properties of interest of this kind of system, for which the dispersion relation is calculated for antisymmetric through to symmetric magnetic field variations.

Production of a non-planar 2DEG can be achieved quite straightforwardly using regrowth technology on previously patterned or etched substrates [13]. Using this technique, it is possible to produce steps, and virtually any other feature in the 2DEG, of controllable height

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and width, enabling investigation of electron motion on shaped surfaces. Application of a uniform magnetic field to these systems results in a non-uniform transverse magnetic field across the 2DEG itself, a technique which offers considerable advantages over current suggestions for the production of magnetic field variations since it allows the production of magnetic steps, barriers, wells [10], and a wealth of other field variations of interest, and in addition enables the use of a variable direction, variable magnitude field.

Considering initially the effect of the curvature of the 2DEG, it is of interest to first obtain the general Hamiltonian for an electron confined to move within a curved surface of thickness d. The expression for an element of length within any surface of finite thickness is given by [14]

$$ds^2 = dr^2 + h_{ij}\, dx^i\, dx^j \qquad i, j = 1, 2$$

here r is the co-ordinate normal to the plane, and x_i, x_j are the two in-plane coordinates. The Hamiltonian for an electron confined to move within this general surface is

$$\hat{H} = -\frac{\hbar^2}{2m^*} h^{-1/2} \left\{ \frac{\partial}{\partial r} h^{1/2} \frac{\partial}{\partial r} + \frac{\partial}{\partial x^i} h^{ij} h^{1/2} \frac{\partial}{\partial x^j} \right\} + V(r)$$

with the confining potential

$$V(r) = \begin{cases} 0 & |r| < d/2 \\ V_b & |r| \geqslant d/2 \end{cases}$$

Here $h = |h_{ij}|$, and the eigenstates and eigenvalues of $\hat{H}$ are Ψ and E. Making the substitution

$$\Psi(r, x_i) = \frac{1}{f}\psi(r, x_i) \qquad \text{where} \qquad f = 4\sqrt{h}$$

gives the general Hamiltonian for a curved surface to be

$$\hat{H} = -\frac{\hbar^2}{2m^*} \left\{ \frac{\partial^2}{\partial r^2} - \frac{1}{f}\frac{\partial^2 f}{\partial r^2} + h^{-1/4} \frac{\partial}{\partial x^i} h^{ij} h^{1/2} \frac{\partial}{\partial x^j} h^{-1/4} \right\}$$

the second term of which would be identically zero in a true 2D system. The above equation is separable, and therefore soluble, if it is assumed that motion normal to the plane is much faster than motion along the plane (the adiabatic approximation) as this allows decoupling of the two components, giving a Hamiltonian for the in-plane motion, and is clearly physically reasonable for the first subband of a 2DEG system.

The model taken for the curved 2DEG is shown in figure 1 and corresponds to a portion of a cylinder joined by two planes, of thickness d. Within the adiabatic approximation, the Hamiltonian for the in-plane part of the wavefunction for this system is

$$\hat{H} = \frac{\pi^2\hbar^2}{8m^* d^2} - \frac{\hbar^2}{2m^*} \left\{ -\frac{1}{R^2}\frac{\partial^2}{\partial \alpha^2} + \frac{\partial^2}{\partial z^2} \right\} - \frac{\hbar^2}{8mR^2} = E_r - \frac{\hbar^2}{2m^*} \left\{ \frac{\partial^2}{\partial l^2} + \frac{\partial^2}{\partial z^2} \right\} - \frac{\hbar^2}{8mR(l)^2}$$

where $l = R\alpha$ is the coordinate parallel to the bending direction and it has been assumed that the walls of the 2DEG are hard. This Hamiltonian is correct to order d^2 but cannot be used when the radius of curvature becomes comparable with the spatial extent of the radial part of the wavefunction [14]. The last term in the Hamiltonian is the one arising from the differentiation of the metric with respect to r, and contains the information about the curvature of the surface. From this term, it can be seen that the general effect of varying the curvature of the space in which electrons move is to produce a change in the potential energy, and for the model under discussion gives rise to a potential well in the curved region, analogous to a central potential†. Considering now the additional effect of applying a magnetic field at some angle γ to the horizontal (see figure 1), then the full Hamiltonian of the model system may be written

$$\hat{H} = -\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial l^2} + V_{\text{mag}} - \frac{\hbar^2}{8m^*R^2} \tag{1}$$

where the magnetopotential is given, in the Landau gauge, by

$$V_{\text{mag}} = \frac{1}{2m^*}\left(\hbar k_z + eA_z\right)^2$$

with

$$A_z = B_{\text{app}}R\{\cos(\gamma+\alpha) - \cos\gamma\} = A_z(\gamma,\alpha) \qquad \text{on the cylindrical region}$$

and

$$A_z = A_z\left(\gamma,\alpha_{\max}\right) - B_{\text{app}}\left(|l| - R\alpha_{\max}\right)\sin\left(\alpha_{\max}\pm\gamma\right) \qquad \text{on the planes.}$$

It is assumed that there are no additional potentials in the z direction, such that the z-momentum operator may be replaced with its eigenvalue. The Hamiltonian for a curved 2DEG subjected to a magnetic field has therefore been reduced to an equation of motion for motion parallel to the bending direction.

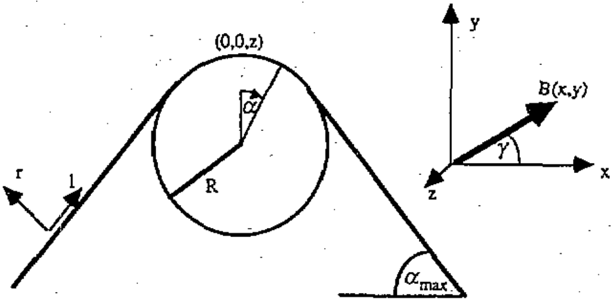

Figure 1. The model for the curved 2DEG corresponds to two planes joined by part of a cylinder. Mutually orthogonal coordinates $(l,\ r,\ z)$ are defined at each point on the surface and the origin of (l, r) is at the apex of the surface. The planes make an angle $\alpha_{\max}$ with the horizontal, and join smoothly to the cylinder, which has a radius R.

† It must be noted at this point that other workers [15, 16] in this field have, using a variety of techniques, obtained a potential due to the curvature of the same magnitude as that derived here, however the sign of the potential is a point of disagreement, we follow here the method described in [14].

The solutions to the equation of motion for an antisymmetric linear variation in the magnetic field ($B = kB_0y$) across a semi-infinite 2DEG have recently been studied in some detail [9], so for simplicity, and for ease of comparison, the case of $\gamma = 0$ will be treated first, i.e. a magnetic field applied parallel to the x-axis to produce a antisymmetric field variation across the 2DEG (see figure 1). However, it must be noted that the system discussed here is more complicated, and in a sense more general than that, for a flat 2DEG, discussed in [9], in that there is also a variation in the magnetic field gradient across the 2DEG, and the field itself can be applied at any angle, such that an antisymmetric field variation is rather a special case.

For the case $\gamma = 0$ the resulting magnetopotential will be either a single potential well centred on $l_0 = 0$, for negative k_z, or a double well potential, the well centres defined by

$$l_0 = R\cos^{-1}\left(1 - \frac{\hbar k_z}{eB_{\mathrm{app}}R}\right) \qquad \text{on the cylinder}$$

or

$$l_0 = \pm\frac{\hbar k_z + eB_{\mathrm{app}}R(\cos\alpha_{\max} + \alpha_{\max}\sin\alpha_{\max})}{eB_{\mathrm{app}}\sin\alpha_{\max}} \qquad \text{on the planes}$$

for positive k_z, i.e. negative velocity states are centred on the apex of the curved 2DEG, and positive velocity states are centred symmetrically either side of the apex, moving towards the edge as k_z increases. The positive velocity states are essentially drifting Landau level states, or edge states (depending upon their location), whereas the central, negative velocity states have no correspondence to any state in the uniform field system. Note that, depending upon the values of R and $\alpha_{\max}$, there is also clearly the possibility of zero-velocity (Landau state) formation on the planar regions, if the region of constant magnetic field is wide enough to contain a cyclotron orbit.

Equation (1) has been solved numerically for a 2000 Å-wide 2DEG formed in GaAs, assuming an effective mass of 0.067 m_e to obtain the dispersion relations of the form shown in figure 2. The dispersion is free-electron-like for negative k_z (apex) states and large positive k_z (edge) states. For positive k_z the double well shape of the magnetopotential results in the formation of linear combinations (symmetric and antisymmetric) of drifting Landau level states until the barrier between the wells is sufficiently high and/or wide that the drifting states become degenerate once more. Following [9], the approximate analytic expressions for the energy and width of the drifting degenerate states, i.e. the states defined by a double well potential centred on $\pm l_0$ are,

$$E = \left(n + \tfrac{1}{2}\right)\frac{\hbar eB_0\sin(l_0/R)}{m^*}$$

$$\Delta l = \left\{\left(n + \tfrac{1}{2}\right)\frac{\hbar}{eB_0\sin(l_0/R)}\right\}^{1/2}$$

giving an analytic expression for the position dependence of the magnetic length

$$l_B = \left(\frac{\hbar}{eB_0\sin(l_0/R)}\right)^{1/2}.$$

It is important to note that, although the magnetic field is creating one-dimensional charge-carrying states, there is no any overall drift velocity within the system until a voltage is applied.

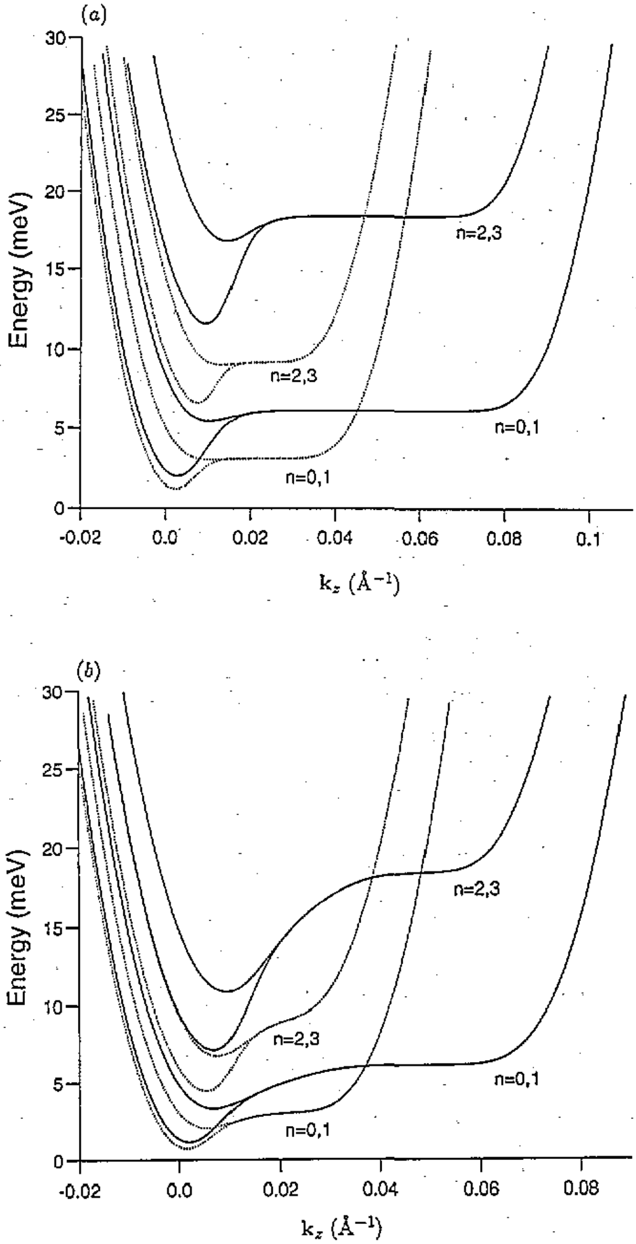

Figure 2. The dispersion relations show the subbands produced as a result of application of a horizontal ($\gamma = 0$) magnetic field to a 2DEG in GaAs (see text), of the form shown in figure 1. Referring to figure 1, the angle which the planes make with the horizontal is taken to be 45°. Both sets, (*a*) and (*b*), of dispersion relations were calculated at an applied field of 10 T (continuous line) and 5 T (dotted line). Regions of negative velocity correspond to states centred on the apex, and regions of positive velocity correspond to states centred on the planes. Note that the velocity can be negative for small positive k_z, due to the presence of the magnetic vector potential. As the radius of curvature is increased from 300 Å (*a*) to 700 Å (*b*), the dispersion across the whole 2DEG increases, and the zero-velocity (flat) regions of the dispersion curves are reduced or disappear. Additionally, increasing the magnetic field increases the range of k_z for which the dispersion curves are flat, due to the reduction in the magnetic length.

Changing the angle of the applied field lifts the degeneracy of the system, resulting in dispersion relations of the form shown in figure 3 (calculated for the same 2DEG system as in figure 2). If the field is applied parallel to the y axis ($\gamma = \pi/2$), the dispersion relation obtained is similar in form to a QHE sample, in that it is symmetric about $k_z = 0$, however, for this system there are two additional sets of current carrying states, one on each side of $l = 0$, with velocities opposing those of the skipping orbits (which travel in opposite directions at opposite edges of this system).

At high magnetic fields, in all the dispersion relations shown, the effect of the bending potential is negligible: the well produced is of order meV or less (in GaAs) in comparison with a subband separation of order 10 meV, however, at low fields it will become more important. As the radius of curvature is decreased the adiabatic approximation will eventually break down, but it is anticipated that the potential produced by the curvature will still be an attractive one, and in this regime will probably be significant even at higher magnetic fields, resulting in electrons 'falling' towards the apex of the structure, producing a wire, or a superlattice-like series of wires for a series of curved 2DEG, even in the absence of a magnetic field.

Transport in a semi-infinite 2DEG subjected to a uniform magnetic field, and in the regime of quantized resistance, is normally discussed in the context of edge channels, since they are the only current-carrying states. Scattering in such systems is only of importance if it results in a change of direction for the electron, that is if the electron is backscattered, which, for a uniform magnetic field, would mean that the electron would have to be scattered from one edge of the sample to the other. The quantum hall regime is one in which backscattering is entirely absent from the system, due to a combination of the strength of the magnetic field and the physical separation of the current-carrying (edge) channels, resulting in the flow of dissipationless current, dissipation occurs only when the Fermi energy passes through the Landau levels. However, for the case of a non-uniform magnetic field, there are current carrying one-dimensional channels located throughout the structure, such that the transport properties will be strongly dependent on the precise shape of the 2DEG, since this determines the form of the dispersion, and the location of the Fermi energy, as this determines which states are involved in the conduction process. For example, if the Fermi energy lies such that it intercepts the dispersion relation at values of k_z corresponding to states located at the apex and edges of the curved 2DEG, then only these states are involved in the conduction process. If a current is driven in the direction using the edge states, then backscattering (to the apex) will be suppressed for a wide enough sample, and/or a high enough magnetic field in precisely the same way as in the QHE. For a fixed Fermi energy, it is anticipated that the onset of scattering will occur at a different magnetic field if the current is passed in the opposite direction, using the apex states, since the width of these states is different from that of the edge states, and in addition, the scattering rates, due to impurities, at the two locations would be expected to be quite different. Driving a current using apex or edge states should not result in a Hall voltage, since although internally there are Hall fields, they are equal and opposite between the apex and the two edges for a symmetric system due to the time-reversal asymmetry.

The non-uniformity in the magnetic field results in a spatial dependence in the magnetic length, and therefore a spatial dependence in the scattering and backscattering probabilities. Accordingly, as the Fermi energy is reduced, (see figure 2) and the current is carried by both edge states and states within the interior of the 2DEG, at some point before the bottom of the subband is reached, there will be the onset of backscattering between the interior states and the apex states. Finally, as the Fermi energy approaches the bottom of the first subband, the current carrying states in this regime are all centred on the apex, and transport

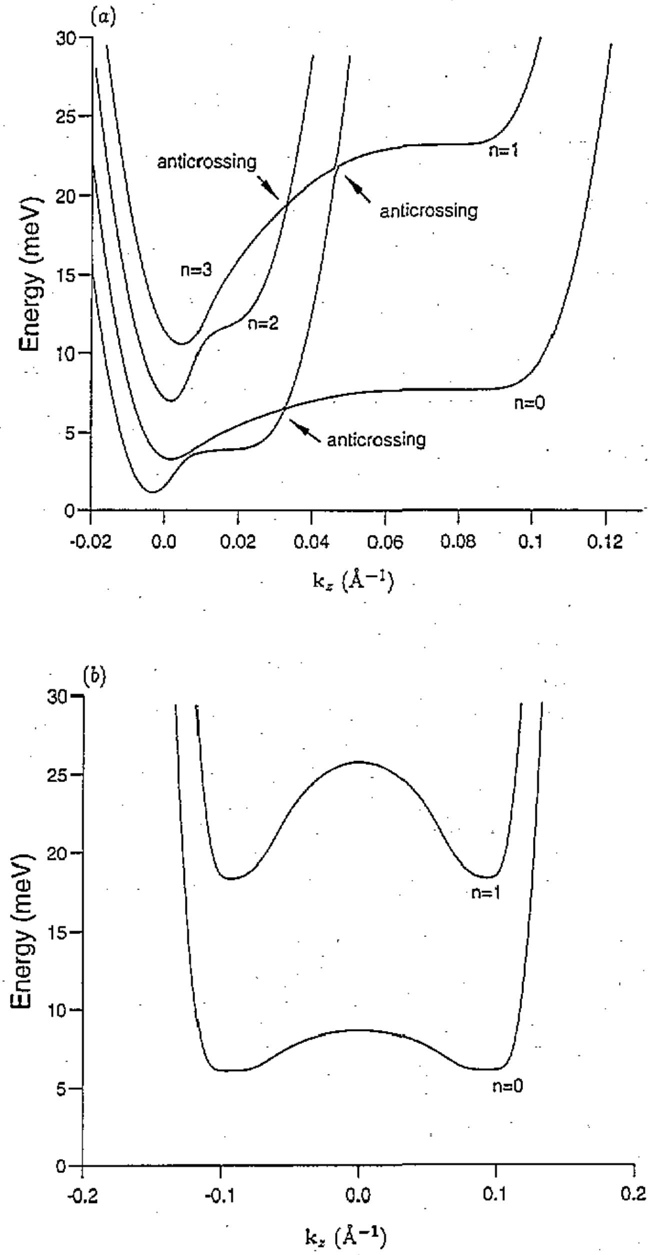

Figure 3. The dispersion relations shown are for the same system as for figure 2 with the radius of curvature fixed at 700 Å and the magnetic field fixed at 10 T (*a*) for a field applied at an angle of 18°, and (*b*) for a field applied at an angle of 90°. In (*a*) it is clearly see that changing the angle of the applied field removes the degeneracy in the system, producing anticrossings in the dispersion relation as a result. In (*b*), the situation is very similar to that found in the quantum hall effect, the difference being that the dispersion relation is not entirely flat in the range of k_z between the two set of edge states, rather there are some additional current carrying states.

is completely one-dimensional, such that it should be possible to use this system to create an isolated, magnetically confined quantum wire. Note that, reduction in the geometrical symmetry of the structure also has the effect of lifting the degeneracy in the dispersion relation, and such systems may be used to produce non-centrally located quantum wires.

It is also of interest to consider what would happen if the (geometric) confinement in the l direction is removed, such that transport in the l direction can be investigated, for example under the influence of magnetic steps, wells or still more complicated structures, such as the barrier/well combination which would arise for the system discussed in this letter.

In summary, a novel method for investigating two-dimensional electron transport in curved space, and in a non-uniform and high magnetic field has been outlined. This method allows investigation and exploitation of the one dimensional structures produced as a result of the application of the magnetic field, and of the curvature of the 2DEG. Magnetic and topological confinement of this form can be combined with electrostatic confinement to create numerous, even more exotic, two-, one- and zero-dimensional structures.

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